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STATISTICS OF SAMPLED RAYLEIGH FADING

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This report addresses the requirements on the length and resolution of realizations of sampled Rayleigh fading used in simulation or hardware testing of communications links. Measurements of the cumulative distribution, mean fade duration, and mean fade separation are compared to ensemble values. It is found that 100 decorrelation time realizations accurately reproduce the statistics of fades which are less than 20 dB, and 400 decorrelation time realizations accurately reproduce the statistics of fades which are less than 30 dB. It is also found that 10 samples per decorrelation time is adequate for 30 dB or less fades as long as interpolation is used between realization points.

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CONVERSION TABLE

Conversion factors for U.S. Customary to metric (SI) units of measurement

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TO GET ◀────────── BY ◀────────── DIVIDE

angstrom	$1.000000 \times E-10$	meters (m)
atmosphere (normal)	$1.01325 \times E+2$	kilo pascal (kPa)
bar	$1.000000 \times E+2$	kilo pascal (kPa)
barn	$1.000000 \times E-28$	meter² (m²)
British thermal unit (thermochemical)	$1.054350 \times E+3$	joule (J)
calorie (thermochemical)	4.184000	joule (J)
cal (thermochemical) / cm²	$4.184000 \times E-2$	mega joule/m² (MJ/m²)
curie	$3.700000 \times E+1$	*giga becquerel (GBq)
degree (angle)	$1.745329 \times E-2$	radian (rad)
degree Farenheit	$t_K = (t_F + 459.67)/1.8$	degree kelvin (K)
electron volt	$1.60219 \times E-19$	joule (J)
erg	$1.000000 \times E-7$	joule (J)
erg/second	$1.000000 \times E-7$	watt (W)
foot	$3.048000 \times E-1$	meter (m)
foot pound force	1.355818	joule (J)
gallon (U.S. liquid)	$3.785412 \times E-3$	meter³ (m³)
inch	$2.540000 \times E-2$	meter (m)
jerk	$1.000000 \times E+9$	joule (J)
joule/kilogram (J/kg) (radiation dose absorbed)	1.000000	Gray (Gy)
kilotons	4.183	terajoules
kip (1000 lbf)	$4.448222 \times E+3$	newton (N)
kip/inch² (ksi)	$6.894757 \times E+3$	kilo pascal (kPa)
ksap	$1.000000 \times E+2$	newton-second/m² (N s/m²)
micron	$1.000000 \times E-6$	meter (m)
mil	$2.540000 \times E-5$	meter (m)
mile (international)	$1.609344 \times E+3$	meter (m)
ounce	$2.834952 \times E-2$	kilogram (kg)
pound force (lbs avoirdupois)	4.448222	newton (N)
pound force inch	$1.129848 \times E-1$	newton-meter (N m)
pound force/inch	$1.751268 \times E+2$	newton/meter (N/m)
pound force/foot²	$4.788026 \times E-2$	kilo pascal (kPa)
pound force/inch² (psi)	6.894757	kilo pascal (kPa)
pound mass (lbm avoirdupois)	$4.535924 \times E-1$	kilogram (kg)
pound mass foot² (moment of inertia)	$4.214011 \times E-2$	kilogram meter² (kg m²)
pound mass/foot³	$1.601846 \times E+1$	kilogram/meter³ (kg/m³)
rad (radiation dose absorbed)	$1.000000 \times E-2$	**Gray (Gy)
roentgen	$2.579760 \times E-4$	coulomb/kilogram (C/kg)
shake	$1.000000 \times E-8$	second (s)
slug	$1.459390 \times E+1$	kilogram (kg)
torr (mm Hg, 0° C)	$1.333220 \times E-1$	kilo pascal (kPa)

*The becquerel (Bq) is the SI unit of radioactivity; 1 Bq = 1 event/s.
**The Gray (Gy) is the SI unit of absorbed radiation.

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SECTION 1

INTRODUCTION

This report is intended to address three questions that arise during simulation or hardware testing activities of communications links under Rayleigh fading conditions: How many decorrelation times per realization are necessary? How many samples per d correlation time are necessary? How should interpolation be done between samples? These questions are answered in part in the DNA signal specification for nuclear scintillation (Wittwer 1980) which requires a minimum of 100 decorrelation times per realization and 10 samples per decorrelation time. However, experience has shown that receiver performance will show considerable statistical variation when the minimum realization length is used. This is particularly true of links which have large power margins and are susceptible to only the very deepest fades. Of course the best way to answer these questions is to measure link performance with realizations of increasing length and resolution until the statistical variation in the results from one realization to the next is acceptable. Unfortunately, the luxury of doing this analysis often does not exist.

The next higher level of analysis of these questions is to look at the statistics of the realizations. This is the approach that will be taken in this report. The first order statistics of the realizations are measured by calculating the cumulative distribution of the amplitudes and comparing this with the Rayleigh distribution. The second order statistics of the realizations are measured by calculating the mean duration and separation of fades and comparing these quantities to ensemble values for Rayleigh fading.

In general, the received signal may be written as the convolution of the channel impulse response function $h(t, \tau)$ with the transmitted signal $s(t)$:

$$u(t) = \int_0^\infty h(t, \tau) s(t - \tau) d\tau \tag{1.1}$$

In either software link simulations or in hardware channel simulators, Equation 1.1 can be implemented as a tapped delay line:

$$u(t) = \sum_{j=0}^{N_D - 1} h(i, j\Delta\tau) s(t - j\Delta\tau) \Delta\tau \tag{1.2}$$

1-1

where N_D is number of taps on the delay line; $\Delta\tau$ is the delay spacing of the delay line; $h(t, j\Delta\tau)$ is the time varying complex weight of the jth tap; and $s(t)$ is the input signal. In a software simulation of link performance, time will also be discretely sampled (i.e. $t = k\Delta t$).

Under Rayleigh fading conditions, $h(t,\tau)$ is a complex, zero mean, normally distributed random variable and thus has a Rayleigh amplitude distribution. It then follows from Equation 1.2 that $u(t)$ is also a complex, zero mean, normally distributed random variable with a Rayleigh amplitude distribution.

A complete analysis of these issues would consider the sampling requirements for each delay of the discrete impulse response function $h(k\Delta t, j\Delta\tau)$. However, this is beyond the scope of this report. Therefore, sampling requirements on the flat fading impulse response function $h(k\Delta t)$, where

$$h(k\Delta t) = \sum_{j=0}^{N_D-1} h(k\Delta t, j\Delta\tau)\Delta\tau \quad , \tag{1.3}$$

will be addressed in this report. The sampling requirements for $h(k\Delta t)$ will give some indication of the sampling requirements for the frequency selective impulse response function $h(k\Delta t, j\Delta\tau)$. Perhaps this should be stated another way: Sampling that is inadequate for $h(k\Delta t)$ will surely be inadequate for $h(k\Delta t, j\Delta\tau)$. Thus it is the intent of this report to define adequate sampling for $h(k\Delta t)$ and to infer adequate sampling requirements for each delay of $h(k\Delta t, j\Delta\tau)$.

SECTION 2

TEMPORAL STATISTICS OF SCINTILLATION

This section is a review of well known results from the classical work of S. O. Rice (1948, 1954, 1958) on the first and second order statistics of Rayleigh fading.

2.1 FIRST ORDER STATISTICS.

Under strong scattering conditions, the electric field incident on the plane of the receiver is the summation of many waves propagating in slightly different directions about the line-of-sight. Under the central limit theorem of statistics, the two orthogonal components of the electric field must then be zero-mean, normally distributed random variables. It is assumed that the two orthogonal components are also independent. The complex narrow-band envelope of the electric field undergoing Rayleigh fading may be then represented as

$$E(t) = x(t) + iy(t) \tag{2.1}$$

where x and y are independent and normally distributed with zero mean standard deviation σ. The carrier frequency term, $\exp(i\omega_0 t)$, has been neglected in this expression. Thus $E(t)$ may be thought of as the output voltage of a down-converter where $x(t)$ is the in-phase component and $y(t)$ is the quadrature-phase component.

It is easy to show that the probability density function of the amplitude of $E(t)$,

$$a(t) = \sqrt{E(t)E^*(t)} = \sqrt{x^2(t) + y^2(t)} \tag{2.2}$$

is Rayleigh:

$$f(a) = \frac{2a}{P_0} \exp\left[\frac{a^2}{P_0} \right] \tag{2.3}$$

where P_0 is the mean power ($P_0 = \langle a^2 \rangle = 2\sigma^2$ where the brackets $\langle \cdot \rangle$ denote an ensemble average) The cumulative distribution of the power P, which is equal to the probability that the power is less than or equal to P, is given by

$$F(P) = \int_0^{\sqrt{P}} f(a)da = 1 - \exp\left[-\frac{P}{P_0}\right] \quad . \tag{2.4}$$

This well known function is plotted in Figure 2.1 versus the ratio P/P_0. It can be seen from the figure that the probability of a 20 dB fade or deeper below the mean power level is 10^{-2} and the probability of a 30 dB fade or deeper is 10^{-3}. This is consequence of the fact that for small values of P/P_0, $F(P) \approx P/P_0$.
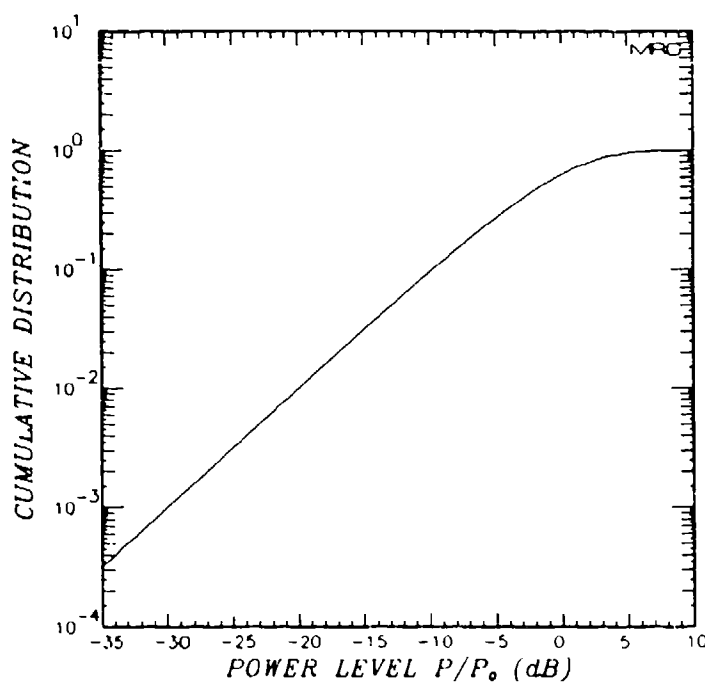

Figure 2.1. Cumulative distribution of Rayleigh fading.

2.2 SECOND ORDER STATISTICS.

The fading rate is determined by the second order statistics of the electric field. The autocorrelation of the electric field is, in general,

$$\langle E(t)E^*(t+\tau)\rangle = \langle x(t)x(t+\tau)\rangle + \langle y(t)y(t+\tau)\rangle = 2\sigma^2\rho(\tau) \quad . \qquad (2.5)$$

There are two limiting forms for the autocorrelation function $\rho(\tau)$. Under strongly disturbed scattering conditions that occur at early times or at the center of the disturbed region, $\rho(\tau)$ has the Gaussian form

$$\rho(\tau) = \exp\left[-\frac{\tau^2}{\tau_0^2}\right] \qquad (2.6)$$

where τ_0, the decorrelation time of the electric field, is defined as the e folding point of the autocorrelation function $(\rho(\tau_0) = e^{-1})$. The corresponding Doppler spectrum of the temporal fluctuations is

$$S(\omega) = \int_{-\infty}^{\infty} \exp(-i\omega\tau)\rho(\tau)d\tau = \sqrt{\pi}\tau_0\exp\left[-\frac{\tau_0^2\omega^2}{4}\right] \qquad (2.7)$$

which also has the Gaussian form. Under less highly disturbed, but still Rayleigh fading conditions, the autocorrelation function is usually assumed to have the form

$$\rho(\tau) = \left[1 + \frac{\alpha|\tau|}{\tau_0}\right]\exp\left[-\frac{\alpha|\tau|}{\tau_0}\right] \qquad (2.8)$$

where the parameter $\alpha(\alpha = 2.146193)$ is determined by the condition that $\rho(\tau_0) = e^{-1}$. The corresponding Doppler spectrum has the form commonly referred to as an f^{-4} spectrum:

$$S(\omega) = \frac{3\tau_0}{\alpha} \frac{1}{\left[1 + \tau_0^2\omega^2/\alpha^2\right]^2} \quad . \qquad (2.9)$$

A comparison of realizations of the impulse response function with Gaussian and f^{-4} Doppler spectrums are shown in Figure 2 where the signal power in dB
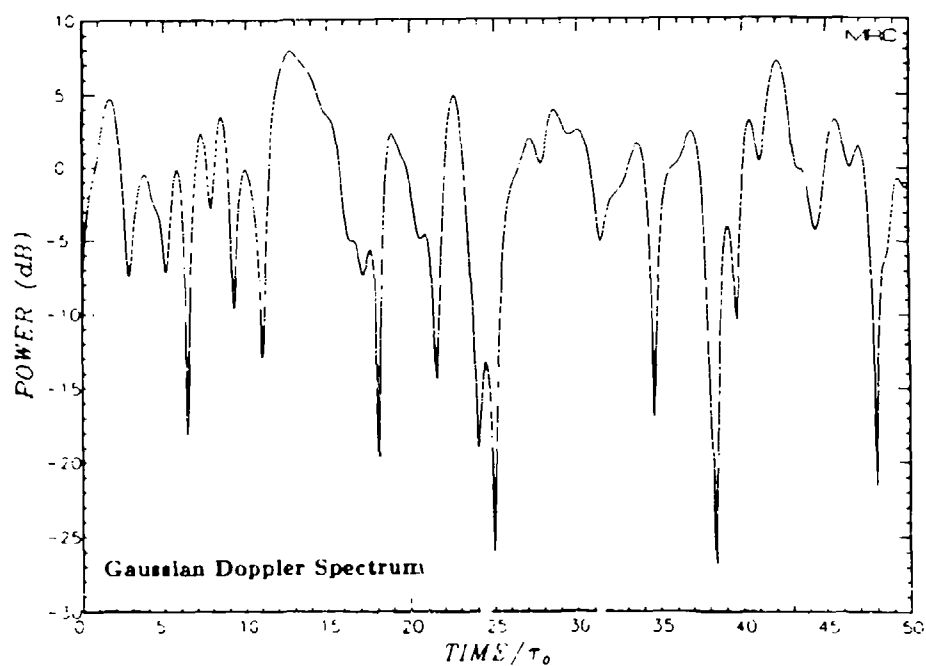
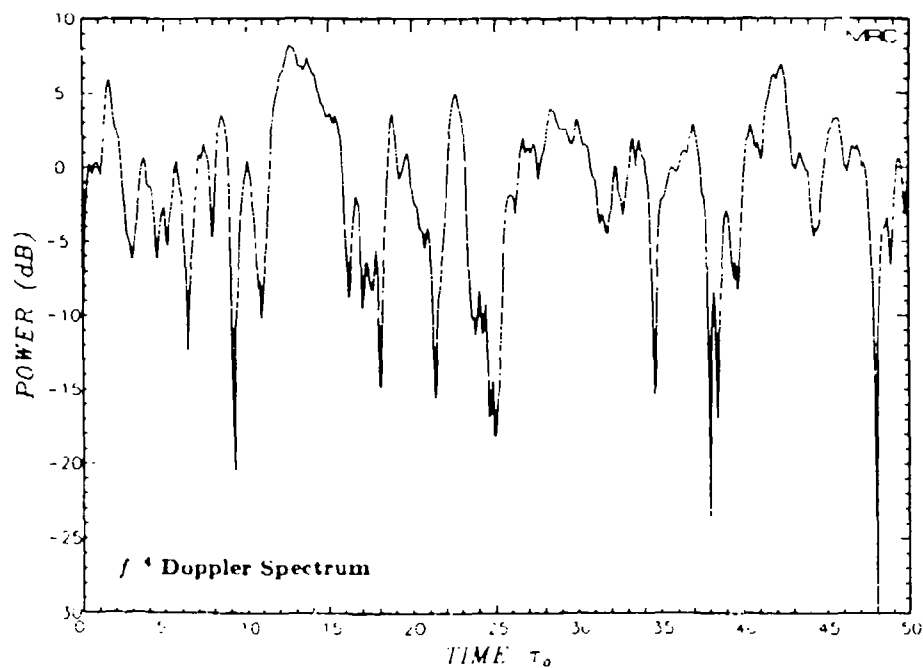Figure 2.2a. Realization with a Gaussian Doppler spectrum.

Figure 2.2b. Realization with an f ^{4} Doppler spectrum.

is plotted versus normalized time/τ_0. These realizations have been generated using statistical techniques described in Appendix B. Both realizations were generated from the same set of random numbers so there is a strong correlation in the features seen in the two realizations. Also, both realizations have unity mean power. The f^{-4} realization has a more spiky appearance due to having more energy at higher Doppler frequencies. The two signals have similar low frequency behavior and the fades of the two signals follow each other quite closely. The difference in the two signals is the high frequency jitter of the f^{-4} signal about the more smoothly varying Gaussian signal. The significance of this on the temporal statistics of the fades will become apparent later.

2.3 TEMPORAL STATISTICS OF RAYLEIGH FADING.

The mean duration and separation of fades below an arbitrary power level P and that of flares above P, are calculated from the mean number $\langle N(P,T)\rangle$ of crossings of the level P in the time interval T.

The probability that the amplitude a will cross the level $\ell = \sqrt{P}$ in the time interval t to $t + dt$ with a positive derivative is equal to the probability that $a' > 0$ and that $\ell - a'dt < a < \ell$. This probability is given by

$$\int_0^\infty da' \int_{\ell - a'dt}^{\ell} da\, f(a,a') = dt \int_0^\infty da'\, a' f(\ell, a')$$

where $f(a,a')$ is the joint probability density function of the amplitude a and its time derivative $a' = da/dt$. The probability that a will cross the level ℓ in the time interval t to $t + dt$ with a derivative of either sign is then

$$dt \int_{-\infty}^{\infty} |a'| f(\ell, a') da' \quad .$$

For stationary processes, the mean number of level crossings of P in the interval t to $t + T$ then becomes

$$\langle N(P,T)\rangle = T \int_{-\infty}^{\infty} |a'| f(\ell, a') da' \quad . \tag{2.10}$$

The joint probability density function of the amplitude and its time derivative is (Rice 1948, Dana 1982)

$$f(a, a') = \left(\frac{2a}{P_0}\right) \exp\left[-\frac{a^2}{P_0}\right] \left(\frac{\tau_0/\Delta}{\sqrt{2\pi P_0}}\right) \exp\left[-\frac{(\tau_0 a'/\Delta)^2}{2 P_0}\right] \tag{2.11}$$

$$(0 \leq a < \infty, -\infty < a' < \infty)$$

This probability density function is derived in Appendix A. It can be seen from the form of Equation 2.11 that the probability density function of a is Rayleigh; that the probability density function of a' is normal with zero mean and with $\Delta^2 P_0/\tau_0^2$ variance; and that a and a' are independent. The functional form of $f(a, a')$ is the same for either of the limiting forms of the Doppler spectrum. The only difference is the value of the parameter Δ which depends on the functional form of the Doppler spectrum ($\Delta = 1$ for the Gaussian spectrum and $\Delta = 1.518$ for the f^{-4} spectrum).

The mean number of level crossings can now be easily evaluated with the result

$$\langle N(P, T) \rangle = \Delta \left(\frac{T}{\tau_0}\right) \sqrt{\frac{8P}{\pi P_0}} \exp\left[-\frac{P}{P_0}\right] \tag{2.12}$$

The effect of the different Doppler spectrums is to scale the mean number of level crossings by the quantity Δ. Recall that this fact was shown qualitatively by comparing the two random realizations in Figure 2. By noting that two level crossings are required to define the beginning and the end of a fade below P, the mean rate of fades below the level P is $\langle N(P, \tau_0) \rangle / 2\tau_0$.

Figure 2.3 shows plots of the mean number of crossings of P in one decorrelation time versus the ratio P/P_0 for the two limiting forms of the Doppler spectrum. The maximum value of $\langle N(P, \tau_0) \rangle$ occurs at $P/P_0 = 1/2$ or -3 dB. As a point of comparison, the median level M of the Rayleigh distribution is $M = P_0 \ell n(2)$ or 1.6 dB below the mean power.

The mean separation $\langle T_S(P) \rangle$ of fades below P is obtained from the mean number of fades per unit time $\beta = \langle N(P, \tau_0) \rangle / 2\tau_0$. For any long time interval T, the mean number of fades is βT and the mean separation is just $T/\beta T$ or $1/\beta$. Thus the mean separation of fades below P is

$$\frac{\langle T_S(P) \rangle}{\tau_0} = \frac{1}{\Delta} \sqrt{\frac{\pi P_0}{2P}} \exp\left[\frac{P}{P_0}\right] \tag{2.13}$$
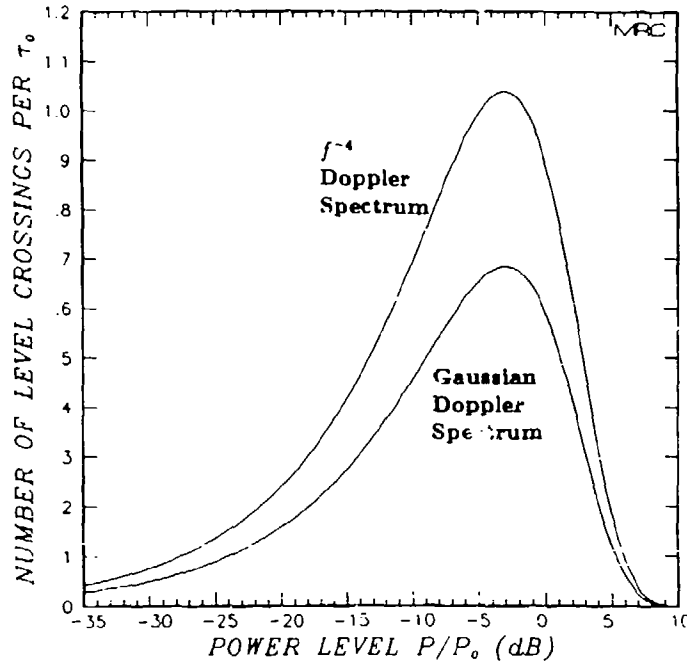
Figure 2.3. Mean number of level crossings per τ_0.

The mean separation of a fade below P is equal to the average time between crossings of P with either a negative value of a' (which defines the start of the fade) or with positive value of a' (which defines the end of the fade). Thus the mean separation of fades below P is also equal to the mean separation of flares above P.

The mean duration $\langle T_{FADE}(P) \rangle$ of fades below P is obtained as follows. During a long time interval T, the total time that the power will be below P is $F(P)T$ where $F(P)$ is the cumulative distribution given in Equation 2.4. The mean duration is then the sum of all durations $F(P)T$ divided by the number of fades βT. The result is

$$\frac{\langle T_{FADE}(P) \rangle}{\tau_0} = \frac{1}{\Delta}\sqrt{\frac{\pi P_O}{2P}}\left\{\exp\left[\frac{P}{P_O}\right] - 1\right\} \quad . \tag{2.14}$$

The mean duration $\langle T_{FLARE}(P) \rangle$ of a flare above P is the mean time that the power stays above P. Using the arguments given above, the mean separation of a fade or a flare is equal to the mean time that the signal is above P plus the mean

time that it is below P : $\langle T_{FADE}(P)\rangle + \langle T_{FLARE}(P)\rangle = \langle T_S(P)\rangle$. The mean duration of a flare is then

$$\frac{\langle T_{FLARE}(P)\rangle}{\tau_0} = \frac{1}{\Delta}\sqrt{\frac{\pi P_O}{2P}} \quad . \tag{2.15}$$

The mean duration and separation of fades and flares are shown in Figure 2.4 for a Gaussian Doppler spectrum. For the f^{-4} Doppler spectrum, all three curves in Figure 2.4 scale by $1/\Delta = 0.6589$.
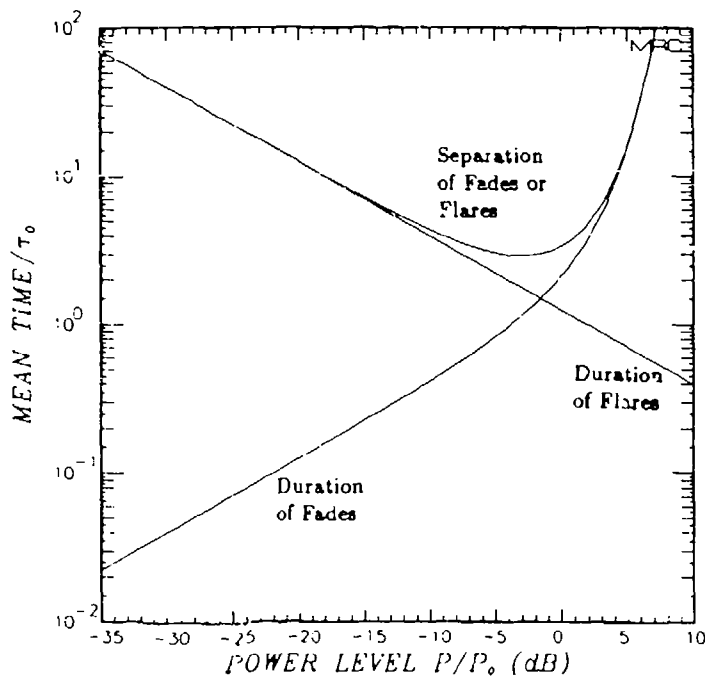

Figure 2.4. Mean duration and separation of fades and flares.

SECTION 3

SAMPLED RAYLEIGH FADING

The requirements on the sampling of Rayleigh fading are given in the DNA signal specification for nuclear scintillation (Wittwer 1980) which requires a minimum of 100 decorrelation times per realization and 10 samples per decorrelation time. The questions that arise from this requirement can be summarized as: How close are such realizations to Rayleigh fading? To address this question, random realizations of Rayleigh fading will be generated; moments of the amplitude, cumulative distribution, and mean fade duration will be measured; and these measured values will be compared with their ensemble values. Of course, because of the finite number of samples in each realization, each of these measurements is, in fact, a random variable with some mean and standard deviation. The variations in these measurements from realization to realization are measured by generating a large number (1024 to be exact) of realizations and computing the mean and standard deviation. A method of generating such realizations is outlined in Appendix B.

The objectives of this section are to present the means and standard deviations of such measurements, and to attempt to answer the above question based on these results. The effects of interpolation between realization samples, and the effects of using longer realizations (more decorrelation times per realization) will also be examined.

3.1 MEASURED STATISTICS OF 100 τ_0 REALIZATIONS.

Realizations generated with $N = 1024$ time samples and $N_0 = 10$ samples per decorrelation time will serve as a baseline calculation. These realizations are also sampled at $\tau_0/10$ time intervals.

One criterion for deciding that a realization has Rayleigh amplitude statistics is that the moments of the amplitude should agree with Rayleigh values. Ensemble values for the moments of the amplitude are easily obtained from Equation 2.3 and are:

$$\langle a \rangle = \frac{1}{2}\sqrt{\pi P_O}$$

$$\langle a^2 \rangle = P_O$$

$$\langle a^3 \rangle = \frac{3}{4}\sqrt{\pi P_0^3} \tag{3.1}$$

$$\langle a^4 \rangle = 2P_0^2 \ .$$

The scintillation index S_4 is the standard deviation of the power:

$$S_4 = \sqrt{\frac{\langle a^4 \rangle - \langle a^2 \rangle^2}{\langle a^2 \rangle^2}} \ . \tag{3.2}$$

It is necessary, but not sufficient, that S_4 equal unity for Rayleigh fading. The scintillation index is a good measure of the statistics of flares but not of fades. Another statistic that is sensitive to the distribution of fades is the mean log amplitude $\overline{\chi}$:

$$\overline{\chi} = \langle \ell n \ a \rangle = \ell n \sqrt{P_0} - \gamma/2 \tag{3.3}$$

where γ is Euler's constant ($\gamma = 0.5772157\cdots$).

The measured mean and standard deviation of the amplitude moments, S_4, and $\overline{\chi}$ are presented in Table 3.1. The measured values of a single realization should equal the ensemble value plus or minus one or two standard deviations. It can be seen from the table that the average values of the moments are close to the ensemble values but that the standard deviations of the higher moments and χ can be as large as 25 percent.

Perhaps a better criterion for the first order statistics is close agreement between the Rayleigh cumulative distribution and the measured cumulative distribution. The measured cumulative distribution (dots plus or minus one-sigma error bars) is plotted in Figure 3.1 along with the ensemble curve (Eqn. 2.4). A level of 0 dB corresponds to the mean power P_0. It can been seen from the figure that 100 τ_0 realizations do indeed have, on the average, a Rayleigh distribution of fades down to at least 30 dB. It is clear, however, that the possible deviation from Rayleigh fading of a single realization becomes larger as one examines deeper fades.

Table 3.1. Amplitude moments of 100 τ_0 realizations.

Moment	Ensemble Value	Average*	Standard Deviation*
$\langle a \rangle$	$\sqrt{\pi P_0}/2$	1.003	0.057
$\langle a^2 \rangle$	P_0	1.005	0.112
$\langle a^3 \rangle$	$3\sqrt{\pi P_0^3}/4$	1.007	0.172
$\langle a^4 \rangle$	$2P_0^2$	1.009	0.243
S_4	1.0	0.983	0.083
$\overline{\chi}$	$-\gamma/2$	0.991	0.221

* Normalized to the ensemble values.

The fidelity of the realizations in reproducing the second order statistics of the fading will be investigated by considering the mean fade duration and separation. The mean fade duration is a good statistic to examine for communications applications because errors often occur in bursts during deep fades. If the fades, on the average, are too long or too short, the error bursts will not have the proper durations and the resulting receiver performance may be misleading. Fade duration measurements and the ensemble curve for a Gaussian Doppler spectrum are shown in Figure 3.2. Because the realizations are generated with 10 samples per decorrelation time, the minimum fade duration is $\tau_0/10$. The measurements reproduce ensemble values for fades down to about 20 dB. Below this level, the average of the measurements is limited to a value of about $\tau_0/10$. The measured value for 30 dB fades is less than $\tau_0/10$ because some 100 τ_0 realizations do not have 30 dB fades. The effect of these realizations on the measurements is to reduce the average values. The measured and ensemble mean fade separation are shown in Figure 3.3. It is likely that 100 τ_0 realizations will not have two fades below the 25 or 30 dB level so a fade separation measurement can not be
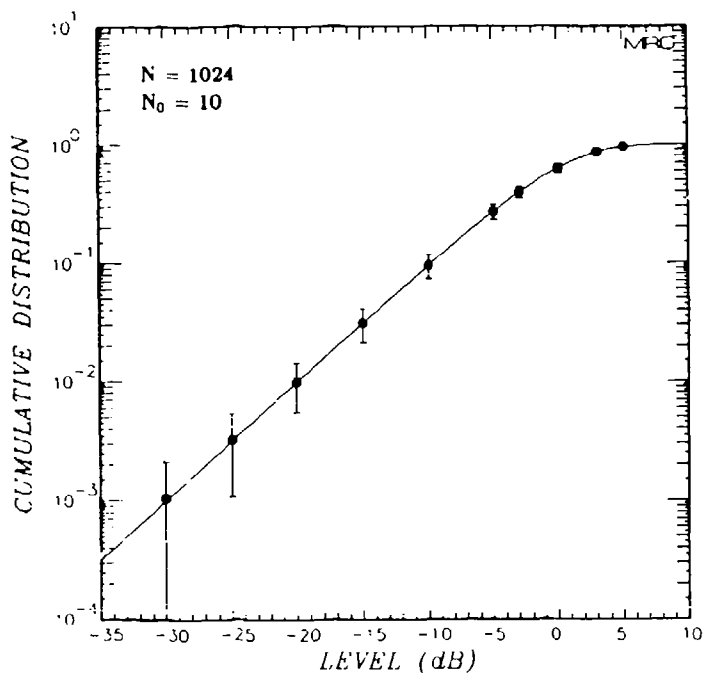
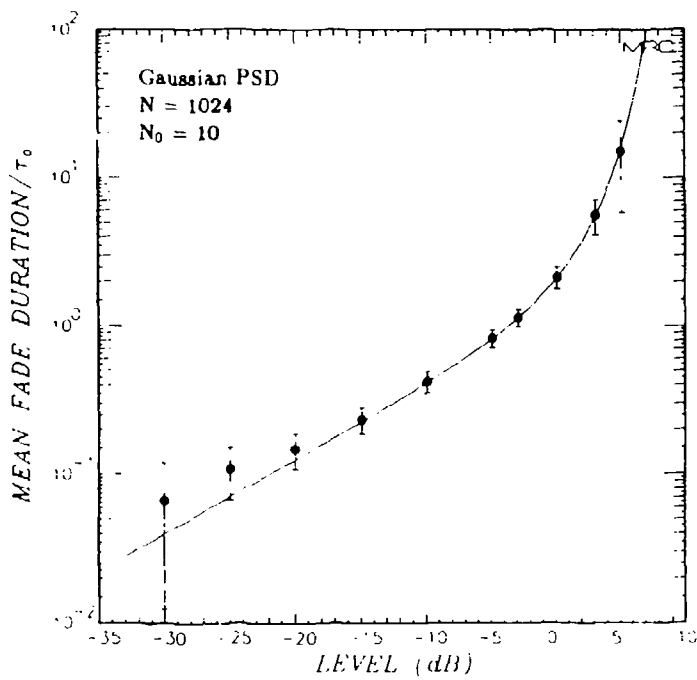Figure 3.1.   Cumulative distribution of 100 τ_0 realizations.

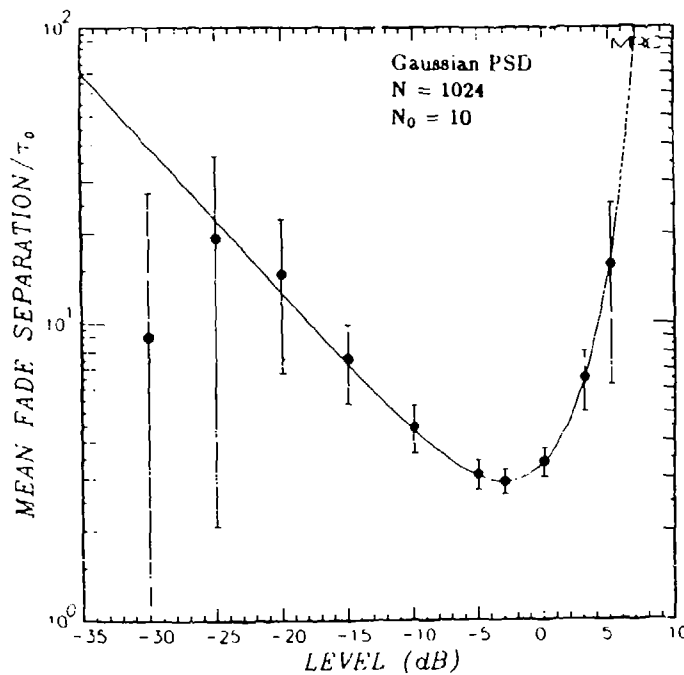Figure 3.2.   Mean fade duration of 100 τ_0 realizations.

Figure 3.3. Mean fade separation of 100 τ_0 realizations.

made. Thus the error bars at these levels are large and the measured separation for 30 dB fades is low by a factor of almost 10.

For fades less than 20 dB or so, the 100 τ_0 realizations reproduce the ensemble values for fade probability, fade duration and fade separation. If deeper fades are of concern, however, then 100 τ_0 realizations sampled at $\tau_0/10$ may not be adequate for producing accurate error rate predictions. Two possible alternatives are discussed below: interpolation and longer realizations.

3.2 EFFECTS OF INTERPOLATION AND LONGER REALIZATIONS.

Although the realizations of the impulse response function are usually generated with samples spaced at $\tau_0/10$, the sampling period of the simulated or tested receiver can be much smaller than $\tau_0/10$. There are at least three approaches to this problem. One approach is to sample the impulse response function at a rate equal to

the sample rate of the receiver. However, if the sample period is much smaller than τ_0, this results in very large realizations if each realization must have 100 decorrelation times. Another approach is to hold the impulse response function constant during the period $\tau_0/10$, and change it abruptly at the end of the period to the next value of the impulse response function. This is, in fact, the approach used to measure the temporal statistics of the realizations, and in principle, this procedure is acceptable because $\tau_0/10$ sampling should result in small changes from one value of the impulse response function to the next. A third approach is to interpolate between values of the impulse response function. Simple linear interpolation will be considered here.

Figure 3.4 shows the mean fade duration measured from exactly the same realizations used to generate the results of the previous section. However, here the realizations are sampled with a period of $\tau_0/40$ using linear interpolation between the $\tau_0/10$ values. During the $\tau_0/40$ period, the impulse response function is held constant. The feature to note in Figure 3.4 is that the measured mean fade durations now lie on top of the theoretical curve for fades down to 30 dB. Of course, if the mean fade
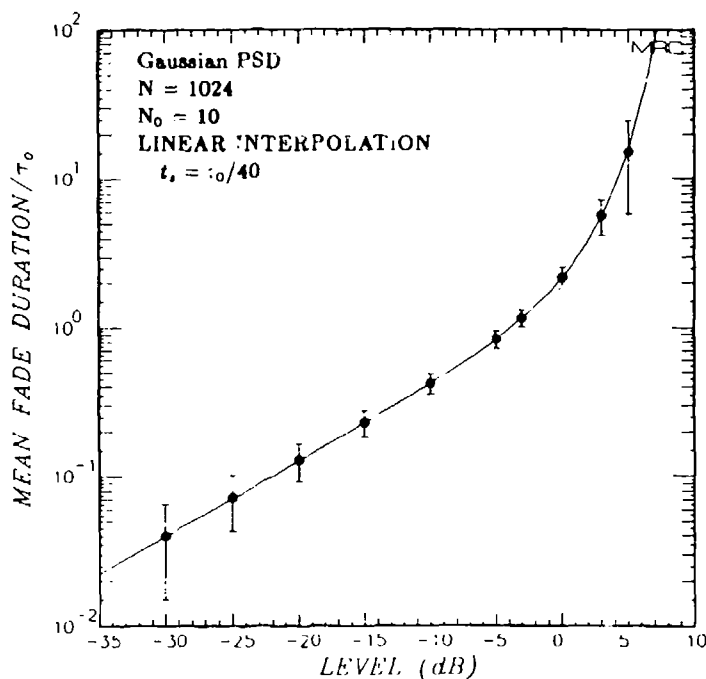

Figure 3.4. Mean fade duration of 100 τ_0 realizations with interpolation.

3-6

duration of 35 dB fades had been measured it would be equal to about $\tau_0/40$ and would lie above the theoretical curve. Because the variations in the mean fade duration from realization to realization are determined by the number of decorrelation times in the realizations rather than the sampling period, the variations around the mean values are about the same size as measured without interpolation.

Finally, the cumulative distribution, mean fade duration, and mean fade separation of 400 τ_0 realizations generated with 10 samples per τ_0 ($N = 4096$, $N_0 = 10$) and sampled at $\tau_0/40$ using linear interpolation are shown in Figures 3.5, 3.6, and 3.7. Here the variations around the mean values are small compared to variations of 100 τ_0 realizations. Also, the mean separation of 30 dB fades for 400 τ_0 realizations is close to the theoretical value indicating that most 400 τ_0 realizations have at least two fades of this depth.

3.3 SUMMARY.

It is clear that 100 τ_0 realizations do not accurately reproduce the duration or separation or fades below about 20 dB. Linear interpolation can be used to accurately reproduce the duration of 20 to 30 dB fades in 100 τ_0 realizations but it takes 400 τ_0 realizations to accurately reproduce the mean separation of 30 dB fades. It is also clear that 10 samples per decorrelation time is sufficient, at least for 30 dB or less fades, as long as interpolation is used between realization points.

In summary, the adequacy of a realization depends on the depth of fades that are of interest. Realizations that are 100 τ_0 long accurately reproduce the duration and separation of fades which are less than about 20 dB. If deeper fades are of interest, then the realizations should be longer and interpolation should be used.

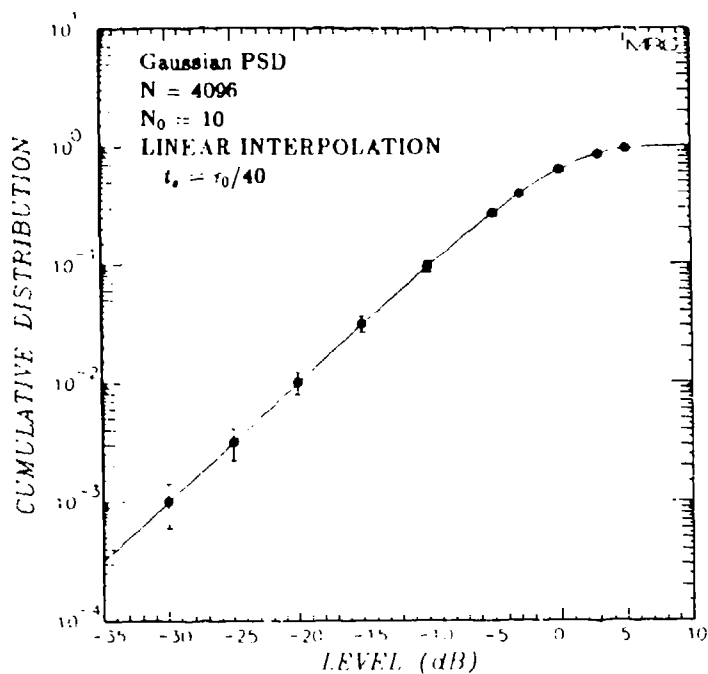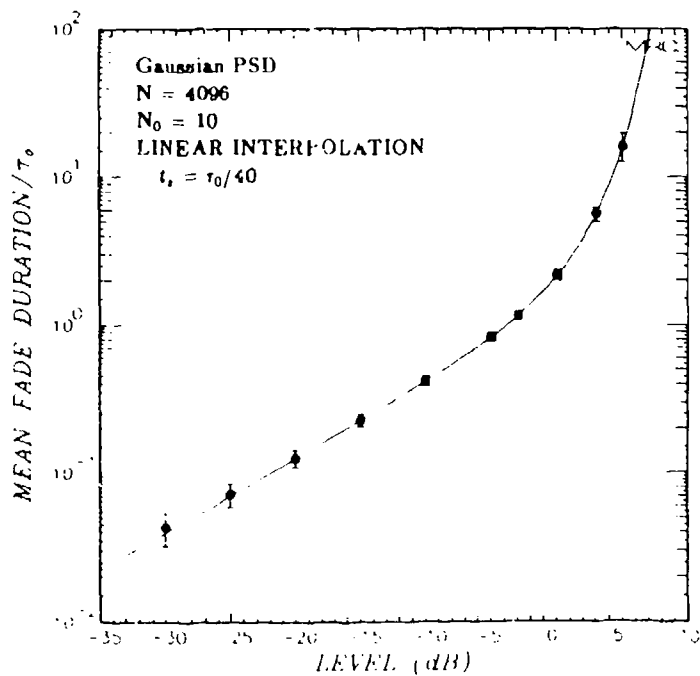**Figure 3.5.** Cumulative distribution of 400 τ_0 realizations with interpolation.

Figure 3.6. Mean fade duration of 400 τ_0 realizations with interpolation.
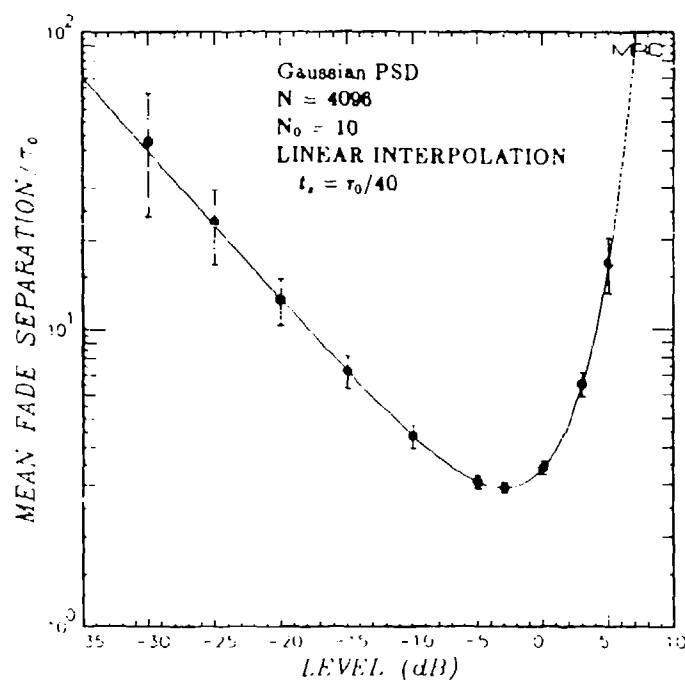
Figure 3.7.   Mean fade separation of 400 τ_0 realizations with interpolation.

SECTION 4

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APPENDIX A

JOINT PROBABILITY DENSITY FUNCTION $f(a,a')$

The purpose of this appendix is to derive the joint probability density function of the Rayleigh amplitude a and its time derivative $a' = da/dt$. This function is required to calculate the temporal statistics of Rayleigh fading. A slightly different form of this derivation was first published by Rice (1948).

The starting point for this calculation is the determination of the joint probability density function of the in-phase and quadrature components x and y of the complex envelope of the electric field. It is assumed that x and y are independent, and, by the central limit theorem, that they are normally distributed. Thus the joint probability density function of x and y is

$$f(x,y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{x^2 + y^2}{2\sigma^2} \right] \quad . \tag{A.1}$$

Now, the probability joint density function of the time derivatives $x' = dx/dt$ and $y' = dy/dt$ must be calculated. It will be shown that x and x' are independent, as are y and y'. It will be assumed that x, x', y, and y' are jointly independent. Thus the joint probability density function of x' and y' is all that is needed in addition to Equation A.1 to write down the joint probability density function $f(x,x',y,y')$. Once this function has been obtained, a simple change of variables from x, x', y, and y' to a and a' will yield the desired function.

In order to determine the distribution of x' (or y'), consider the random function $x(t)$ written as a Fourier stochastic integral

$$x(t) = \int_{-\infty}^{\infty} z(\omega) \exp(i\omega t) \frac{d\omega}{2\pi} \quad . \tag{A.2}$$

The quantity $z(\omega)$ is a random function in the Doppler domain. It is useful to assume that $z(\omega)$ is a zero-mean, normally distributed random process, although this is not necessary because the central limit theorem will make $x(t)$ normally distributed for almost any reasonable distribution of $z(\omega)$. However, with the normal assumption for $z(\omega)$, $x(t)$ is the sum of many independent, normally distributed random variables, and is necessarily a zero-mean, normally distributed random variable.

Before continuing, it is interesting to show the relationship between the random spectral components $z(\omega)$ and the Doppler spectrum $S(\omega)$. The autocorrelation function of the stationary process $x(t)$ may be written as

$$\rho(t_2 - t_1) = \frac{\langle x(t_1) x(t_2) \rangle}{\sigma^2} \tag{A.3}$$

$$= \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \frac{\langle z(\omega_1) z^*(\omega_2) \rangle}{\sigma^2} \exp(i\omega_1 t_1 - i\omega_2 t_2) \quad .$$

However, the correlation function $\rho(\tau)$ may also be written in terms of $S(\omega)$:

$$\rho(\tau) = \int_{-\infty}^{\infty} S(\omega) \exp(i\omega\tau) \frac{d\omega}{2\pi} \quad . \tag{A.4}$$

Note that, in general, $S(\omega)$ must be an even function if the autocorrelation function $\rho(\tau)$ is real. In order to ensure that the integral in Equation A.3 is only a function of time difference $\tau = t_1 - t_2$, the integrand must contain a factor $2\pi\delta(\omega_1 - \omega_2)$. Using the Dirac delta function to collapse the double integral in Equation A.3 and comparing the result with Equation A.4 gives

$$\langle z(\omega_1) z^*(\omega_2) \rangle = 2\pi\sigma^2 \, \delta(\omega_1 - \omega_2) S(\omega_1) \quad . \tag{A.5}$$

This equation also demonstrates that the random Doppler spectral components of $z(\omega)$ are uncorrelated, which is a consequence of the assumption that the random process $x(t)$ is stationary.

The time derivative of $x(t)$ is given by differentiating Equation A.2, with a similar expression holding for y':

$$x'(t) = \int_{-\infty}^{\infty} (i\omega) z(\omega) \exp(i\omega t) \frac{d\omega}{2\pi} \quad . \tag{A.6}$$

Because $z(\omega)$ is normally distributed with zero mean, $x'(t)$ will also be normally distributed with zero mean. The variance of $x'(t)$ is

$$\langle x'(t) x'(t) \rangle \quad \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \omega_1 \omega_2 \langle z(\omega_1) z^*(\omega_2) \rangle \exp i(\omega_1 - \omega_2) t$$

$$= \sigma^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^2 S(\omega) \tag{A.7}$$

$$= -\sigma^2 \frac{d^2\rho(\tau)}{d\tau^2}\bigg|_{\tau=0} \quad .$$

In general, the variance of $x'(t)$ may be written as

$$\langle x'(t)x'(t)\rangle = \frac{2\sigma^2\Delta^2}{\tau_0^2} \tag{A.8}$$

where

$$\Delta = \begin{bmatrix} 1.0 & \text{(Gaussian Doppler Spectrum)} \\ \alpha/\sqrt{2} = 1.5176 & (f^{-4}\text{Doppler Spectrum}) \end{bmatrix} \tag{A.9}$$

and where the parameter α is defined in Equation 2.8. The cross correlation of $x'(t)$ and $x(t)$ is

$$\langle x'(t)x(t)\rangle = \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi}(i\omega_1) \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \langle z(\omega_1)z^*(\omega_2)\rangle \exp[i(\omega_1 - \omega_2)t]$$

$$= -i\sigma^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega \, S(\omega) = 0 \quad . \tag{A.10}$$

Equations A.2 and A.5 and the fact that $S(\omega)$ is an even function have been used in reducing Equations A.7 and A.10. Because $x(t)$ and $x'(t)$ are uncorrelated and normally distributed, they are also independent. Identical results hold for the variance of y' and the cross correlation of y and y'.

The joint probability density function of x, x', y, and y' may now be written down:

$$f(x,x',y,y') = \left(\frac{1}{2\pi\sigma^2}\right)\exp\left[-\frac{x^2 + y^2}{2\sigma^2}\right]\left(\frac{\tau_0^2}{4\pi\sigma^2\Delta^2}\right)\exp\left[-\frac{\tau_0^2(x'^2 + y'^2)}{4\sigma^2\Delta^2}\right] \quad . \tag{A.11}$$

This function may be transform to the desired function of a and a' by making the change of variables

$$x = a\cos\theta \tag{A.12}$$

$$y = a\sin\theta \quad.$$

The time derivatives of x and y are

$$x' = a'\cos\theta - a\theta'\sin\theta \tag{A.13}$$

$$y' = a'\sin\theta + a\theta'\cos\theta \quad.$$

which gives the usual polar coordinate equations

$$x^2 + y^2 = a^2 \tag{A.14}$$

$$x'^2 + y'^2 = a'^2 + a^2\theta'^2 \quad.$$

The probability density function coordinate transformation is

$$f(x,x',y,y')dxdx'dydy' = f(a,a',\theta,\theta')|\det(J)|dada'd\theta d\theta' \tag{A.15}$$

where the determinate of the Jacobian of the transformation is

$$\det(J) = \det\begin{bmatrix} \dfrac{\partial x}{\partial a} & \dfrac{\partial x'}{\partial a} & \dfrac{\partial y}{\partial a} & \dfrac{\partial y'}{\partial a} \\ \dfrac{\partial x}{\partial a'} & \dfrac{\partial x'}{\partial a'} & \dfrac{\partial y}{\partial a'} & \dfrac{\partial y'}{\partial a'} \\ \dfrac{\partial x}{\partial \theta} & \dfrac{\partial x'}{\partial \theta} & \dfrac{\partial y}{\partial \theta} & \dfrac{\partial y'}{\partial \theta} \\ \dfrac{\partial x}{\partial \theta'} & \dfrac{\partial x'}{\partial \theta'} & \dfrac{\partial y}{\partial \theta'} & \dfrac{\partial y'}{\partial \theta'} \end{bmatrix} \tag{A.16}$$

$$= \det\begin{bmatrix} \cos\theta & -\theta'\sin\theta & \sin\theta & \theta'\cos\theta \\ 0 & \cos\theta & 0 & \sin\theta \\ -a\sin\theta & -a'\sin\theta - a\theta'\cos\theta & a\cos\theta & a'\cos\theta - a\theta'\sin\theta \\ 0 & -a\sin\theta & 0 & a\cos\theta \end{bmatrix}$$

$$= a^2 \quad.$$

The joint probability density function of a and a' is obtained by integrating over θ and θ':

$$f(a, a') = \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} d\theta' f(a, a', \theta, \theta') \quad , \tag{A.17}$$

with the result

$$f(a, a') = \left(\frac{a}{\sigma^2}\right) \exp\left[-\frac{a^2}{2\sigma^2}\right] \left(\frac{\tau_0}{2\pi^{1/2}\Delta\sigma}\right) \exp\left[-\frac{\tau_0^2 a'^2}{4\Delta^2\sigma^2}\right] \quad . \tag{A.18}$$

Thus it is apparent from this equation that the probability density function of a is Rayleigh; that the probability density function of a' is normal with zero mean and variance of $2\Delta^2\sigma^2/\tau_0^2$; and that a and a' are independent because their joint probability density function is separable into a function of a times a function of a'.

APPENDIX B

REALIZATIONS OF RAYLEIGH FADING

The methods of generating random realizations of the impulse response function for Rayleigh fading channels are discussed in detail elsewhere (Knepp 1982 or Dana 1986) and are only briefly reviewed in this appendix. The starting point of the method of generating a realization of flat Rayleigh fading is the Doppler Power Spectral Density (PSD) function, $S(\omega)$. This function has two commonly used forms which were discussed in Section 2:

$$S(\omega) = \sqrt{\pi}\,\tau_0 \exp\left[-\frac{\tau_0^2 \omega^2}{4}\right] \qquad \text{(Gaussian PSD)} \qquad (B.1)$$

and

$$S(\omega) = \frac{4\tau_0}{\alpha} \frac{1}{\left[1 + \tau_0^2 \omega^2/\alpha^2\right]^2} \qquad \text{(f}^{-4}\text{ PSD)} \qquad (B.2)$$

where $\alpha = 2.146193$ in the f^{-4} form of the PSD. The quantity $S(\omega)d\omega/2\pi$ is the mean fraction of power in the Doppler frequency interval $\omega/2\pi$ to $(\omega + d\omega)/2\pi$.

The discrete realizations of the channel impulse response function will contain N time samples with N_0 samples per decorrelation time. Thus the time spacing of the discrete samples is

$$\Delta t = \frac{\tau_0}{N_0} \quad , \tag{B.3}$$

and the total time duration of the realization is $N\Delta t$. In the Doppler frequency domain, the spacing of the discrete samples is

$$\Delta\omega = \frac{2\pi}{N\Delta t} \quad . \tag{B.4}$$

Note that the quantity $\Delta\omega\Delta t$, which will appear later in a Fourier transform, is equal to $2\pi/N$.

The samples in the frequency domain are generated by first calculating the fraction of signal power in each Doppler frequency bin, $S_j = S(j\Delta\omega)\Delta\omega/2\pi$. For the Gaussian PSD,

$$S_j = \frac{\sqrt{\pi}N_0}{N}\exp\left[-\frac{j^2\pi^2N_0^2}{N^2}\right] \quad , \qquad (j = -N/2,\cdots,N/2-1) \qquad (\text{B}.5)$$

and for the f^{-4} PSD,

$$S_j = \frac{4N_0}{\alpha N}\left[1+\frac{j^2 4\pi^2 N_0^2}{\alpha^2 N^2}\right]^{-2} \quad . \qquad (j = -N/2,\cdots,N/2-1) \qquad (\text{B}.6)$$

Next, the random Doppler frequency spectrum $H(j\Delta\omega)$ of the impulse response function is generated:

$$H(j\Delta\omega) = \frac{2\pi}{\Delta\omega}\sqrt{S_j}\,\xi_j \quad . \qquad (\text{B}.7)$$

The leading factor $2\pi/\Delta\omega$ has been included so that the discrete Fourier transform of $H(j\Delta\omega)$ will be dimensionless. The random components of the spectrum, ξ_j, are complex, normally disturbed random variables with the properties:

$$\langle \xi_j \xi_k^* \rangle = \delta_{j,k} \qquad (\text{B}.8)$$

$$\langle \xi_j \xi_k \rangle = 0 \quad , $$

where the quantity $\delta_{j,k}$ is the Kronecker delta symbol. The random samples of ξ may be generated using

$$\xi_j = \sqrt{-\ln(u_{1,j})}\exp(2\pi i u_{2,j}) \qquad (\text{B}.9)$$

where $u_{1,j}$ and $u_{2,j}$ are independent random variables uniformly distributed on the interval $[0,1)$.

Finally, the random Doppler spectrum of the channel impulse response function is Fourier transformed to the time domain. In continuous notation, its Fourier transform is

$$h(t) = \int_{-\infty}^{\infty} H(\ ) \exp(i\omega t)\frac{d\omega}{2\pi} \quad , \tag{B.10}$$

and in discrete notation

$$
\begin{aligned}
h(k\Delta t) &= \sum_{j=-N/2}^{N/2-1} H(j\Delta\omega)\exp[i(j\Delta\omega)(k\Delta t)]\frac{\Delta\omega}{2\pi} \\
&= \sum_{j=-N/2}^{N/2-1} \sqrt{S_j}\,\xi_j\exp[i2\pi jk/N] \quad ,
\end{aligned}
\tag{B.11}
$$

where $k = 0, 1, \cdots, N-1$.

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